UNIT 8. MOTION



## 1. WHAT IS MOTION?

**Kinematics** is the science of describing the motion of objects using **words**, **diagrams**, **numbers**, **graphs**, and **equations**. The goal of any study of kinematics is to develop models which serve to describe (and ultimately, explain) the motion of real-world objects.

Movement is one of the most common phenomena which occur around us, even in nature we find movements which are easily observed and truly beautiful .

**Everything is in motion**. For example, we are all moving around the sun. Can you give more examples of movement?

But, the motion of an object with respect to another object is nothing but relative motion.

For example, if you are **traveling on a train** then **you are at rest with respect to the observer on the train**, but you are in **motion with respect to the person outside the train**. This is called relative motion.

**Motion is a change in position of a body with respect to a point or a set of coordinates (reference frame) that we consider at rest.**

**This means that when we talk about motion, we must look at it relative to something else.**

To describe motion, we use rates such a **velocity**, **speed**, and **acceleration**.

## 2. POSITION AND TRAJECTORY

**Position**

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame.

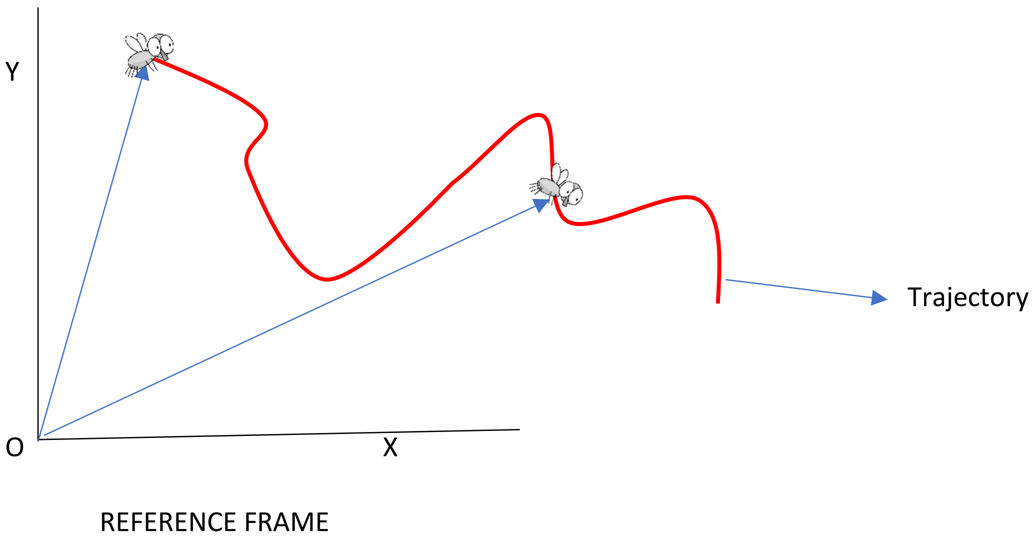
A **reference frame** is a point or a set of coordinates that can be used to determine positions and velocities of objects in that frame.

Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame.

For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board.

In other cases, we use reference frames that are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame.

**The position of a body is the place where it is with respect to a reference frame. It is given in terms of the distance, in a straight line, between the body and the point taken as the origin of the reference frame.**

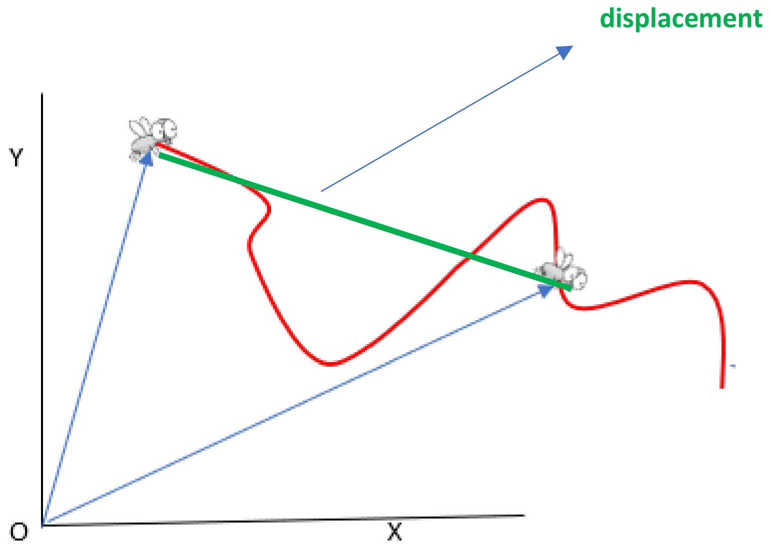


If we follow the movement of a fly, we observe that it changes its position over time, an goes from the initial point to the final one. The fly has changed its position with respect to the origin of the frame of reference. We also observe that it follows a path in its movement, represented by the red line.

The **trajectory** of a movement is the geometric line that a body takes while it moves. It can be straight or a curve.

## 3. DISPLACEMENT AND DISTANCE TRAVELLED

**Displacement** is the distance is a straight line, which separates two positions of a moving objects in different moments in time.

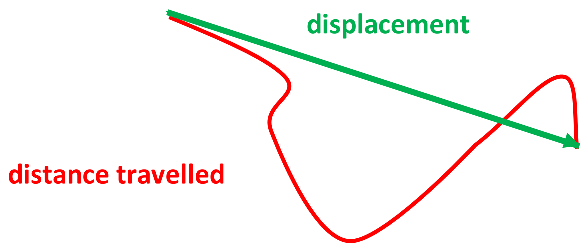



REFERENCE FRAME

Depending on the trajectory, the distance travelled may be different between the same initial and final positions. If the fly moves in a straight line, the distance travelled would different than if it goes wandering around.

The **distance travelled (s)** is the length a moving object goes measured on the  trajectory.

In the case of this fly, distance travelled and displacement are different.



# 4. TIME, SPEED AND VELOCITY

## SPEED

There is more to motion than distance and displacement.

Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts, such as speed or velocity.

If someone asked you what of these two animals is faster, you would have no problem in answering.

When we compare the speed of two bodies in motion, we have to take into account two variables: the distance they travel and the time they take to do so.

What's the difference between two identical objects traveling at different speeds?

We know that the one moving faster (the one with the greater speed) will go farther than the one moving slower in the same amount of time.

**Speed is the rate at which a body changes its position**

### Average speed

Whatever speed is, it involves both distance and time. "Faster" means either "farther" (greater distance) or "sooner" (less time).

- Speed is directly proportional to distance when time is constant.
- Speed is inversely proportional to time when distance is constant.

Combining these two rules together gives the definition of speed in symbolic form.

$$v_m = \frac{s}{t}$$

**Average speed, $v_m$, is the relationship (or quotient) between the distance travelled and the time taken to do it.**

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{time taken}}$$

**Units of speed:**

If we look at the equation for average speed, we can deduce that speed is always given in units of length divided by units of time

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{time taken}} \xrightarrow{\text{SIU}} \frac{m}{s}$$

According to the SI, the unit of length is the metre and the unit of time is the second. So, the SI unit of speed is metre per second (m/s), although speed is also given in other units, like km/h or cm/s.

## Instantaneous speed


During a typical trip, your car will undergo a series of changes in its speed. If you were to inspect the speedometer readings at regular intervals, you would notice that it changes often.

The speedometer of a car reveals information about the instantaneous speed of your car. It shows your speed at a particular instant in time.

**Instantaneous speed, v, or just speed, is the speed of an object at a particular moment.**

The instantaneous speed of an object is not to be confused with the average speed.

Average speed is a measure of the total distance travelled in a given period of time; it is sometimes referred to as the distance *per* time ratio.

Suppose that during your trip to school, you travelled a distance of 10 km and the trip lasted 0.2 hours (12 minutes). The average speed of your car could be determined as:

$$Average\ speed = \frac{Distance\ travelled}{time\ taken} \longrightarrow v_m = \frac{10\ km}{0{,}2\ h}$$

On the average, your car was moving with a speed of 50 kilometres per hour. During your trip, there may have been times that you were stopped and other times that your speedometer was reading 65 km/h. Yet, on average, you were moving with a speed of 50 km/h.

If a moving object always moves at the same speed, moves with constant speed, the instantaneous and average speeds coincide. In this case the movement is **uniform.**

## VELOCITY

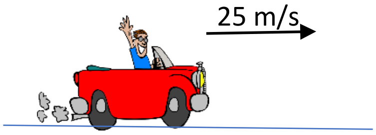
As with speed, the SI unit of velocity is the metre per second (m/s).

**Velocity** tells you the **speed** at which an object is travelling, but it also tells you the **direction** of travel:

$$\boldsymbol{average\ velocity = \frac{distance\ moved\ in\ a\ particular\ direction}{time\ taken}}$$

**Instantaneous velocity, v: it is the velocity at a given instant.**

Instantaneous velocity, v, specifies the velocity of an object each instant in time. If a body has constant velocity it does not speed up, slow down or change direction. Its average velocity is the same no matter what time interval during the motion it is measured.




In diagrams, you can show a velocity with an arrow 25 m/s: $\longrightarrow$

Or you can use a + or – to give the direction:

- + 25 m/s (25 m/s to the right)
- -25 m/s (25 m/s to the left)

**WHAT THEY MEAN**

| | |
|---|---|
| A steady speed of 25 m/s | A distance of 25 m is travelled every second |
| A steady velocity of + 25 m/s | A distance of 25 m is travelled every second (to the right) |
| A steady velocity of -25 m/s | A distance of 25 m is travelled every second (to the left) |

Quantities like velocity which have a direction as well as a value are called **vector quantities**. Vector quantities include: acceleration, force, weight.

Quantities which are defined only with a value, and don´t have a particular direction are called **scalar**. Scalar quantities include: temperature, mass, time, energy.

## 5. UNIFORM LINEAR MOTION (ULM)

**Uniform linear motion refers**

- ✓ **Linear** motion: motion along a straight line.
- ✓ **Uniform** motion: motion with constant velocity, the instantaneous velocity do not change over time.

**Uniform linear motion** is motion with a straight-line trajectory over the same distance in the same amount of time, that is, the speed and the velocity are constant over the whole trajectory.

If a body has constant velocity it does not speed up, slow down or change direction. Its average velocity is the same no matter what time interval during the motion it is measured.

**A uniform linear motion (ULM) means that it has constant velocity (meaning that the speed, direction and sense do not change over time)**

## Equations for ULM

In all ULM, the instantaneous velocity is always the same, so we can say that this value and the average velocity are the same.

$$\boxed{v = v_m = \frac{s}{t}}$$

Using this equation, we can obtain other mathematical equations which allows us to calculate distance travelled or time taken:

$$\boxed{t = \frac{s}{v}} \qquad \boxed{s = v \times t}$$

## ULM GRAPHS

We can learn a lot from motion graphs. They can tell us how far something has travelled, how fast it is moving and all the speed changes there have been.

## Distance-time graphs

Distance-time graphs show how far an object has moved at different times.

**Distance-time graph**

**Independent variable. Plotted on the x-axis**

**Dependent variable**

**Plotted on the y-axis**

The distance-time graph of a uniform linear motion is a straight line which can passes through the origin or not.

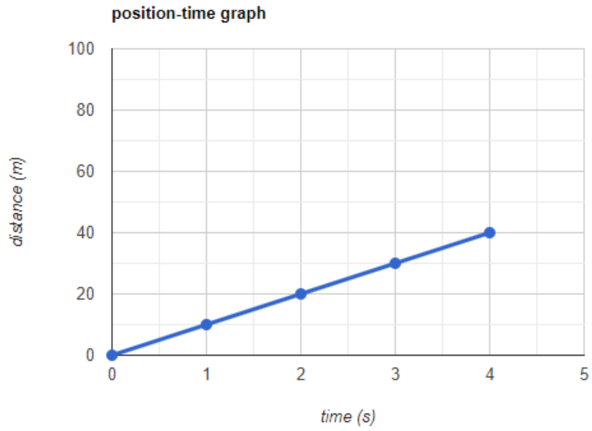
Imagine a car travelling along a road. We can measure the position of the car (the distance to the origin) every  second.

Distance and time are recorded in a chart, and used to plot a graph.

**Steady speed of 10 m/s** 

**GRAPH 1**

| Time (s) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Position (m) | 0 | 10 | 20 | 30 | 40 |




Line rises 10 m on distance scale for every 1 s on time scale.

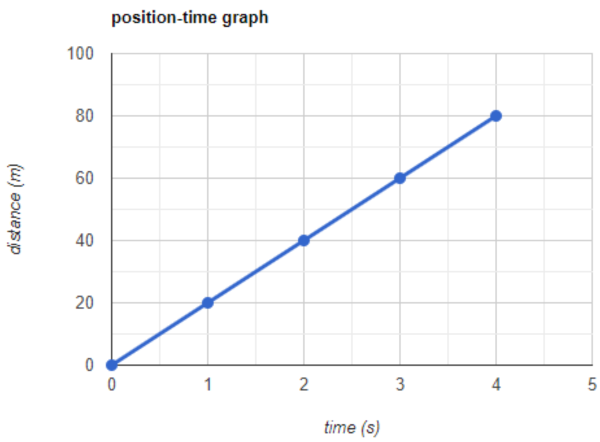
The same distance travelled in the same time.

Velocity is constant.

**Steady speed of 20 m/s**

**GRAPH 2**

| Time (s) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Position (m) | 0 | 20 | 40 | 60 | 80 |




Line is sleeper than before. It rises 20 m on distance scale for every 1 s on time scale.

Again, the same distance travelled in the same time.

Velocity is constant, and greater than in the first example.

If you imagined these lines to be hills, you would say that the line from graph 2 is steeper **than the** line from graph 1. Line 2 has a **greater slope** than line 1.

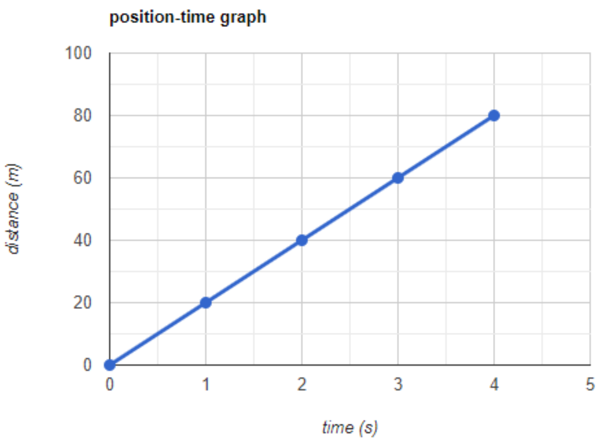
**The slope of a position-time graph gives information about an object's velocity.**

For example, a **small slope means a small velocity**; a **negative slope means a negative velocity**; a **constant slope (straight line) means a constant velocity**; a **changing slope (curved line) means a changing velocity**.

Thus, the shape of the line on the graph (straight, curving, steeply sloped, mildly sloped, etc.) is descriptive of the object's motion.

Even more, the slope of the line on a position-time graph is equal to the velocity of the object. If the object is moving with a velocity of +4 m/s, then the slope of the line will be +4 m/s. If the object is moving with a velocity of -8 m/s, then the slope of the line will be -8 m/s. If the object has a velocity of 0 m/s, then the slope of the line will be 0 m/s.

## Determining the Slope on a position-time graph




Let´s consider the position-time graph on the right:

**The line is loping upwards to the right.**

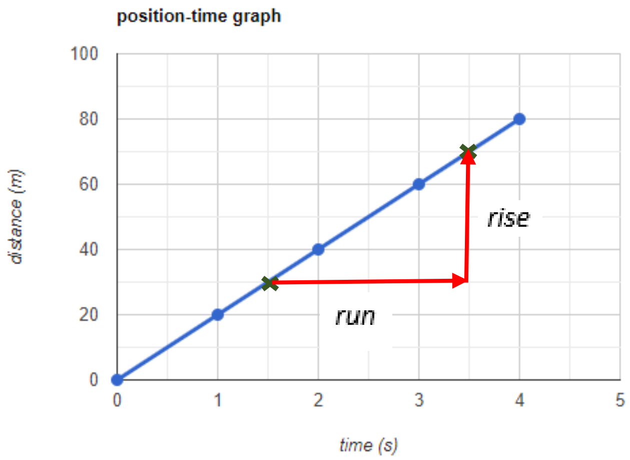
But, mathematically, by how much does it slope upwards for every 1 second along the horizontal (time) axis?

To answer this question, we must use the slope equation.

$$\boxed{slope = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}}$$

## Using the Slope Equation

The slope equation says that the slope of a line is found by determining the amount of rise of the line between any two points divided by the amount of run of the line between the same two points. In other words,




- Pick two points on the line and determine their coordinates.
- Determine the difference in y-coordinates of these two points ( *rise* ).
- Determine the difference in x-coordinates for these two points ( *run* ).
- Divide the difference in y-coordinates by the difference in x-coordinates (rise/run or slope).

The diagram below shows this method being applied to determine the slope of the line. Note that three different calculations are performed for three different sets of two points on the line. In each case, the result is the same: the slope is 20 m/s (positive).

For points: (1,5 s, 30 m) and (3,5 s, 70 m)

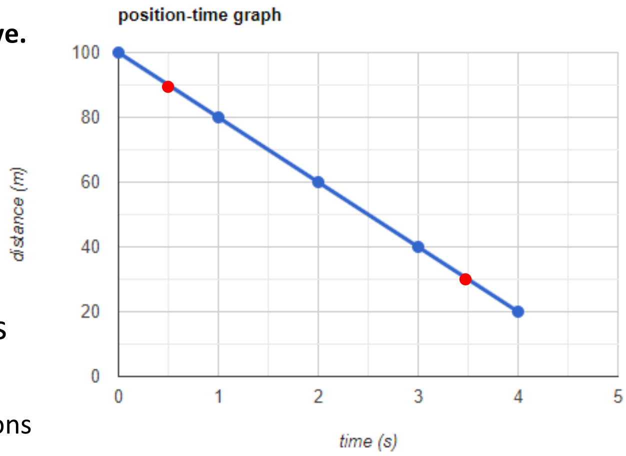
$$slope = \frac{\Delta y}{\Delta x} = \frac{70\ \text{m} - 30\ \text{m}}{3{,}5\ \text{s} - 1{,}5\ \text{s}} = \frac{40\ \text{m}}{2\ \text{s}} = 20\ \text{m/s}$$

For points: (1 s, 20 m) and (4 s, 80 m)

$$slope = \frac{\Delta y}{\Delta x} = \frac{80\ m - 20\ m}{4\ s - 1\ s} = \frac{60\ \text{m}}{3\ \text{s}} = 20\ \text{m/s}$$

Let´s consider now a more difficult example:

**The line is loping downwards to the right. The slope is negative.**




Let´s determine the velocity (I.e. the slope) of the object as displayed by the graph.

For points (0,5 s , 90 m) and (3,5 s and 30 m)

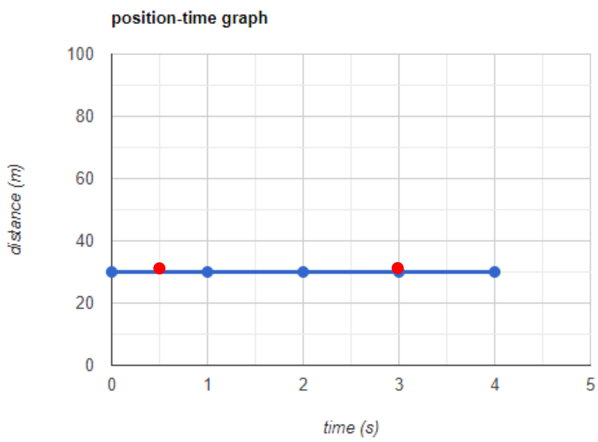
$$slope = \frac{\Delta y}{\Delta x} = \frac{30\ m - 90\ m}{3{,}5\ s - 0{,}5\ s} = \frac{-\ 60\ \text{m}}{3\ \text{s}} = -\ 20\ \text{m/s}$$

You can try to calculate the slope for any two other combinations of points, and the result will be the same.

The slope is negative (negative velocity).

**Whenever an object or body moves in a positive direction described by the coordinate system, its velocity is considered positive. Its velocity is termed negative if it goes in the opposite direction.**

**Horizontal position-time graph**



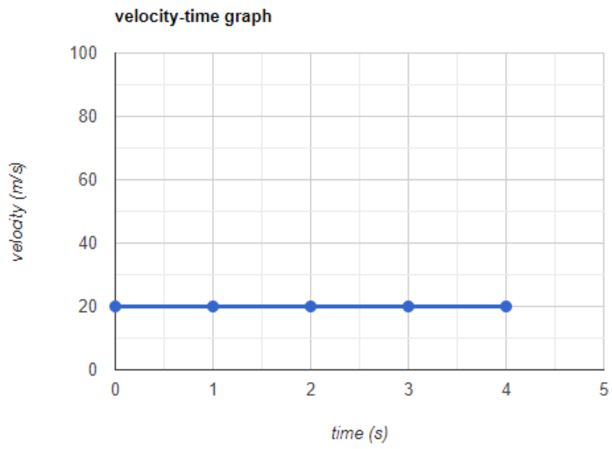

On a straight-line graph like this on the right, a horizontal line, the slope is 0.

The object do not move from 30 m.

$$slope = \frac{\Delta y}{\Delta x} = \frac{30\ m - 30\ m}{3s - 0{,}5\ s} = 0$$

## LUM velocity-time graphs

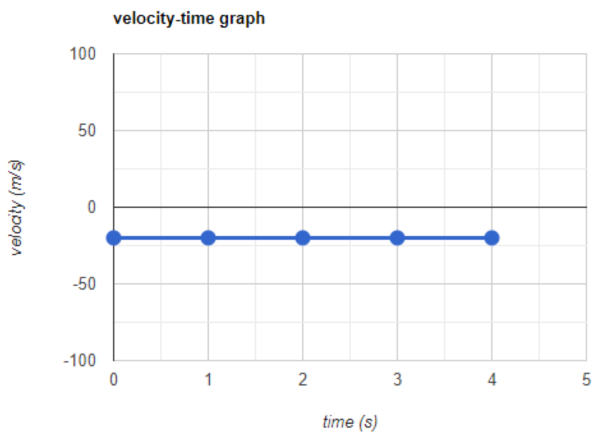
Look at the velocity-time graphs on the right. This object travels at a constant velocity of 10 m/s (+10 m/s) for 4 seconds.




The velocity-time graph for a LUM is a horizontal straight line.

In this case, the velocity is constant and positive, +20 m/s, so, the object is moving to the right (or upwards if the movement is vertical).

Here is another example, but now velocity is negative, -20 m/s and constant. This is a LUM graph for an object moving with -20 m/s.




## 6. ACCELERATION

From rest, a rally car can reach a velocity of 100 km/h in 2,5 s or less. It gains velocity very rapidly. It has a high acceleration.

Like velocity, acceleration has a direction, it is a vector quantity.

Acceleration is calculated as follows:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

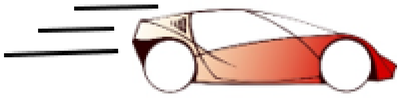
For example, if a car gains an extra 40 m/s of velocity in 10 seconds:

$$acceleration = \frac{40\ \text{m/s}}{10\ \text{s}} = 4\ \text{m/s per second} \longrightarrow 4\ \text{m/s}^2$$

**Acceleration is the physical quantity that describes changes in the velocity of an object. It is a vector quantity, ant he SI unit is m/s$^2$**

An object is said to be accelerating if it is :

- Speeding up
- Slowing down
- Changing direction
- Speeding up and changing direction
- Slowing down and changing direction



$$acceleration = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\text{final velocity-initial velocity}}{\text{time taken}} = \frac{\Delta v}{\Delta t}$$

The acceleration of an object tells us how quickly its velocity is changing. The more the velocity of an object changes is a certain time, the greater the acceleration.

**Acceleration can be positive or negative**. If a car has an acceleration of -4 m/s$^2$, it is losing 4 m/s of speed every second. A negative acceleration is sometimes called **deceleration.**

## 7. UNIFORMLY ACCELERATED LINEAR MOTION (UALM)

Uniformly accelerated linear motion refers:

- ✓ Uniformly accelerated: the numerical value of acceleration is constant, so the value of the velocity changes at the same rate.
- ✓ Linear: the trajectory is a straight line, so, the direction of the acceleration is constant.

A **uniformly accelerated linear motion** means that the acceleration is constant, the magnitude and direction do not change over time.

### <u>Equations for UALM</u>

As we have just seen, the acceleration in an UALM is always the same, and as a consequence, the velocity value changes.

- **Velocity time equation**:

From the mathematical equation of the acceleration:

$$acceleration = \frac{\Delta v}{\Delta t} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}} = \frac{vf\text{-}vi}{t}$$

$$v_f - v_i = a \times t$$

$$\boxed{v_f = v_i + (a \times t)}$$

$v_f$ = final velocity (or v)

$v_i$ = initial velocity (or $v_0$)

a= acceleration

t = time

- **Position time equation:**

Position for an uniformly accelerated linear motions can be determined using the equation:

$$\boxed{x = x_i + v_i \times t + ½\ (a \times t^2)}$$

x = final position

$x_i$ = initial position ($x_0$)

$v_i$ = initial velocity ($v_0$)

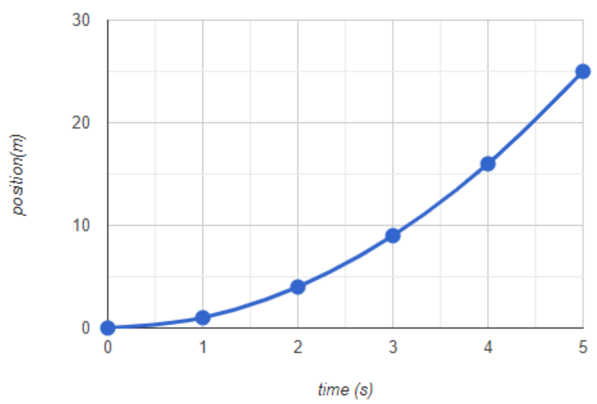
a= acceleration

t = time

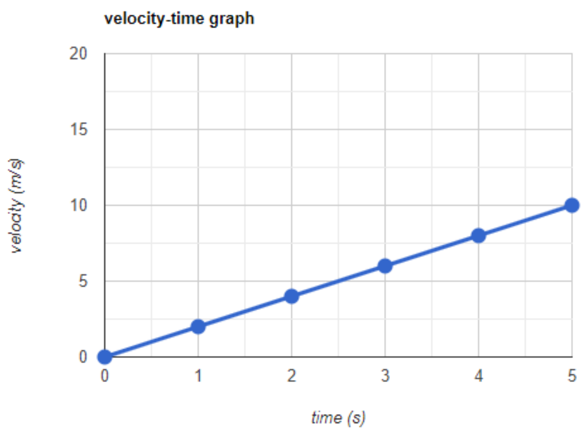
## Graphs for UALM

These are the graphs for a body moving from rest( $v_i$ = 0 m/s) and from initial position = 0 m (from the origin of the frame of reference), with constant acceleration= 2 m/s$^2$.

Position-time graph




velocity-time graph



- **Position-time graph:**

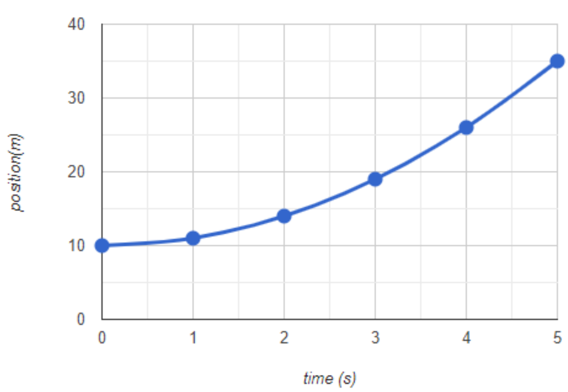
The position-time graph in a uniformly accelerated motion is a **parabola**, that does not necessarily have to start from the origin

If you remember the equation for position:

$$x = x_0 + v_i \times t + \frac{1}{2}(a \times t^2)$$

Position (x) is in a quadratic proportion to time (t), and so, we get a curve called **parabola** when we represent the position-time graph for a uniformly accelerated motion.

Notice that in the graph on the right, the initial position is not = 0. Initial position , $x_i$ is 10 m.




| Time (s) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Position (m) | 10 | 11 | 14 | 19 | 26 | 35 |

- **velocity-time graph:**

We can guess that the position-time graph for a uniformly accelerated linear motion is a **straight inclined line**. Remember that UALM has constant acceleration, the value of the velocity changes at the same rate.

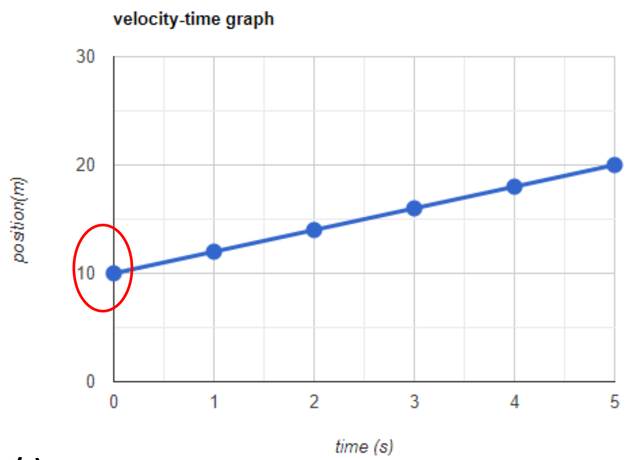
$$v = v_i + (a \times t)$$

| **Time (s)** | **0** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|
| **Velocity (m/s)** | **10** | **12** | **14** | **16** | **18** | **20** |

Velocity is directly proportional to time.

We can calculate the slope of this straight line

$$slope = \frac{\Delta y}{\Delta x} = \frac{18\ m/s - 12\ m/s}{4s - 1\ s} = \frac{6\ m/s}{3\ s} = 2\ m/s^2$$




**The slope of a velocity-time graph gives information about an object's acceleration.**

**Equation for this movement:**

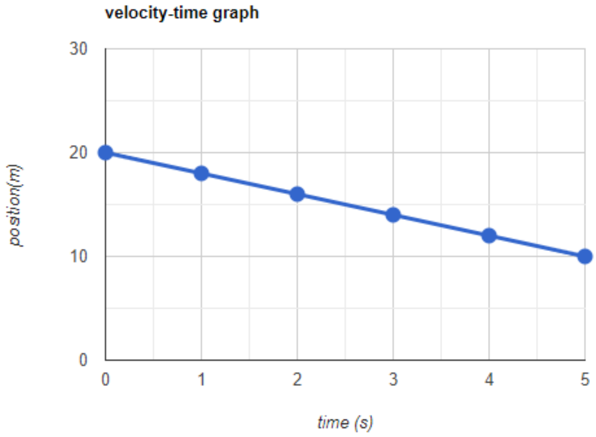
**v= 10 + 2 × t (m/s) → v= 10 + 2t (m/s)**

You can see in the data table and the velocity-time graph on the right that now the velocity of the object is decreasing.

| Time (s) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Velocity (m/s) | 20 | 18 | 16 | 14 | 12 | 10 |

We can calculate the acceleration:

$$a = -2\ m/s^2$$



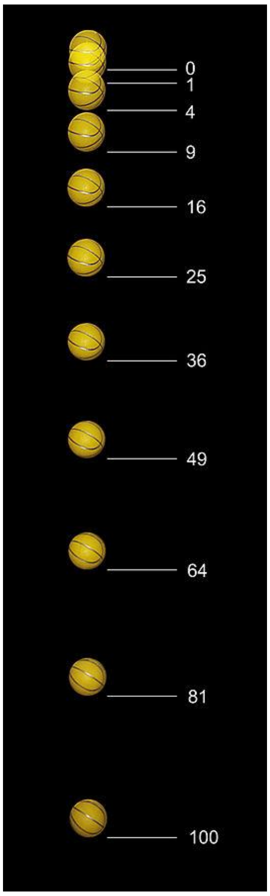

In fact, the velocity is changing by a constant amount, - 2 m/s in each second of time.

$$v = 20 - 2t\ (m/s)$$

## Vertical motion due to gravity: free fall

Free fall is a specific case of UALM.

Naturally, objects fall. This is due to gravity of the Earth. If you drop your pen, it starts from an initial position, at rest, and as it falls, we can observe that it moves faster. There is an increase in velocity during the fall, the pen is accelerating.




In **free fall**, all objects are moving with a **constant acceleration of -9,8 m/s²**, which we usually represent with the letter **g** (because it is due to gravity).

This negative sign represents only the direction of acceleration due to gravity. So, 'g' is negative in free fall because the direction of 'g' is opposite to the positive direction defined as positive (in the vertical, positive upwards and negative downwards).

The picture shows an initially-stationary ball which is allowed to fall freely under gravity, and drops a distance which is proportional to the square of the elapsed time

Free fall equations:

$$v = v_i + (a \times t) \longrightarrow v = v_i + (-9{,}8)\ t\ m/s$$

$$x = x_i + v_i \times t + ½\ (a \times t^2) \longrightarrow y = y_i + v_i \times t + ½\ (-9{,}8)\ t^2$$

y is the symbol we generally use to represent vertical position