Exercicios de apoio

Exercicio nº 1.-

Acha **a)** $\int\left(3-\frac{2}{3}x+\frac{1}{4}x^2-0{,}05x^3\right)dx$; **b)** $\int\left(5x^4-3x^2+2x\right)dx$

Exercicio nº 2.-

Descubre **a)** $\int\left(9x^2+5x+3\right)dx$; **b)** $\int\left(8e^x-\frac{5}{x}+9x\right)dx$

Exercicio nº 3.-

Acha **a)** $\int 5x\sqrt[3]{1-9x^2}\,dx$; **b)** $\int \frac{7x}{4+3x^2}dx.$

Exercicio nº4.-

Calcula **a)** $\int 7e^{3x+4}dx$; **b)** $\int 3e^{-x}dx.$

Exercicio nº5.-

Calcula $\int_0^1 \frac{(2x-1)^2}{4x^2+1}dx.$

Exercicio nº6.-

Calcula o valor de a>0nos seguintes casos **a)** $\int_0^3 \frac{1}{x+1}dx=a$; **b)** $\int_0^a \frac{1}{x+1}dx=3$; **c)** $\int_0^3 \frac{1}{x+a}dx=5.$

Exercicio nº7.-

Calcula a área do recinto plano acoutado limitado pola gráfica de $f(x)=xe^{x^2}$ para $x \geq 0$, o eixe OX e a recta x=2

Exercicio nº8.-

Calcula a área do recinto limitado polas curvas y=$\sqrt{2x}$ e y = $\frac{x^2}{2}$

Exercicio nº9.-

Acha a área do recinto plano delimitado por y=x^2+1 , y=3

Exercicio nº10.-

Calcula as seguintes integrais **a)** $\int_{\pi/4}^{\pi/3} \frac{\cos x}{\operatorname{sen} x}dx$; **b)** $\int_0^1 \frac{e^{2x}+e^x}{\left(e^x+1\right)^2}dx.$

Exercicios resoltos:

Exercicio nº 1.-

a) $\int\left(3-\frac{2}{3}x+\frac{1}{4}x^2-0{,}05x^3\right)dx=3x-\frac{x^2}{3}+\frac{x^3}{12}-0{,}0125x^4+k;$ b) $\int\left(5x^4-3x^2+2x\right)dx=x^5-x^3+x^2+k.$

Exercicio nº 2.-

a) $\int\left(9x^2+5x+3\right)dx=3x^3+\frac{5x^2}{2}+3x+k;$ b) $\int\left(8e^x-\frac{5}{x}+9x\right)dx=8e^x-5\ln x+\frac{9x^2}{2}+k.$

Exercicio nº 3.-

a) $$\int 5x\sqrt[3]{1-9x^2}\,dx=\left\{\begin{array}{l}u=1-9x^2\\ u'=-18x\Rightarrow dx=\dfrac{du}{-18x}\end{array}\right\}=\int 5x\cdot u^{1/3}\,\frac{du}{-18x}=-\frac{5}{18}\int u^{1/3}du=-\frac{5}{24}u^{4/3}+k=\frac{-5\left(1-9x^2\right)^{4/3}}{24}+k;$$

b) $$\int\frac{7x}{4+3x^2}dx=\left\{\begin{array}{l}u=4+3x^2\\ u'=6x\Rightarrow dx=\dfrac{du}{6x}\end{array}\right\}=\int\frac{7x}{u}\cdot\frac{du}{6x}=\frac{7}{6}\int\frac{du}{u}=\frac{7}{6}\ln u+k=\frac{7}{6}\ln\left(4+3x^2\right)+k.$$

Exercicio nº 4.-

a) $\int 7e^{3x+4}dx=\frac{7}{3}e^{3x+4}+k;$ b) $\int 3e^{-x}dx=-3e^{-x}+k.$

Exercicio nº 5.-

$$\int\frac{(2x-1)^2}{4x^2+1}dx=\int\frac{4x^2+1-4x}{4x^2+1}dx=\int\left(1-\frac{4x}{4x^2+1}\right)dx=x-\frac{1}{2}\ln\left(4x^2+1\right)\Rightarrow\int_0^1\frac{(2x-1)^2}{4x^2+1}dx=x-\frac{1}{2}\ln\left(4x^2+1\right)\Big|_{x=0}^{x=1}=1-\frac{\ln 5}{2}.$$

Exercicio nº 6.-

a) $\int_0^3\frac{1}{x+1}dx=a\Rightarrow \ln(x+1)\Big|_{x=0}^{x=3}=\ln 4=a;$ b) $\int_0^a\frac{1}{x+1}dx=3\Rightarrow\ln(x+1)\Big|_{x=0}^{x=a}=\ln(a+1)=3\Rightarrow a+1=e^3\Rightarrow a=e^3-1;$

c) $$\int_0^3\frac{1}{x+a}dx=5\Rightarrow\ln(x+a)\Big|_{x=0}^{x=3}=\ln(3+a)-\ln a=\ln\frac{3+a}{a}=5\Rightarrow\frac{3+a}{a}=e^5\Rightarrow 3+a=a\cdot e^5\Rightarrow 3=a\cdot e^5-a=a\left(e^5-1\right)\Rightarrow$$

$$\Rightarrow a=\frac{3}{e^5-1}.$$

Exercicio nº 7.-

$$f(x)=xe^{x^2}\Rightarrow f(x)=0\Rightarrow x=0\Rightarrow A=\left|\int_0^2 xe^{x^2}dx\right|\Rightarrow F(x)=\frac{e^{x^2}}{2}\Rightarrow\begin{cases}F(0)=\dfrac{1}{2}\\F(2)=\dfrac{e^4}{2}\end{cases}\Rightarrow A=\frac{e^4-1}{2}u^2.$$

Exercicio nº 8.-

$$h(x)=\frac{x^2}{2}-\sqrt{2x}\Rightarrow h(x)=0\Rightarrow\frac{x^2}{2}=\sqrt{2x}\Rightarrow x^4-8x=0\Rightarrow x\left(x^3-8\right)=0\Rightarrow x=0,2\Rightarrow A=\left|\int_0^2 h(x)dx\right|\Rightarrow$$

$$\Rightarrow H(x)=\frac{x^3}{6}-\frac{\sqrt{8x^3}}{3}\Rightarrow\left.\begin{cases}H(0)=0\\H(2)=-\dfrac{4}{3}\end{cases}\right\}\Rightarrow A=\frac{4}{3}u^2.$$

Exercicio nº 9.-

$$h(x)=x^2+1-3=x^2-2\Rightarrow h(x)=0\Rightarrow x=\pm\sqrt{2}\Rightarrow A=\left|\int_{-\sqrt{2}}^{\sqrt{2}} h(x)dx\right|\Rightarrow H(x)=\frac{x^3}{3}-2x\Rightarrow\left.\begin{cases}H\left(-\sqrt{2}\right)=\dfrac{4\sqrt{2}}{3}\\H\left(\sqrt{2}\right)=-\dfrac{4\sqrt{2}}{3}\end{cases}\right\}\Rightarrow$$

$$\Rightarrow A=\left|H\left(\sqrt{2}\right)-H\left(-\sqrt{2}\right)\right|=\frac{8\sqrt{2}}{3}u^2.$$

Exercicio nº10.-

a) $$\int_{\pi/4}^{\pi/3}\frac{\cos x}{senx}dx=\ln\left(senx\right)\Big|_{x=\frac{\pi}{4}}^{x=\frac{\pi}{3}}=\ln\frac{\sqrt{3}}{2}-\ln\frac{\sqrt{2}}{2}=\ln\frac{\sqrt{3}}{\sqrt{2}}=\frac{1}{2}\ln\frac{3}{2};$$

b) $$\int\frac{e^{2x}+e^x}{\left(e^x+1\right)^2}dx=\int\frac{e^{2x}+e^x}{e^{2x}+2e^x+1}dx=$$

$$=\left\{\begin{array}{l}u=\left(e^x+1\right)^2=e^{2x}+2e^x+1\\u'=2\left(e^{2x}+e^x\right)\Rightarrow dx=\dfrac{du}{2\left(e^{2x}+e^x\right)}\end{array}\right\}=\int\frac{e^{2x}+e^x}{u}\cdot\frac{du}{2\left(e^{2x}+e^x\right)}=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln u=\frac{1}{2}\ln\left(e^x+1\right)^2=\ln\left(e^x+1\right)\Rightarrow$$

$$\Rightarrow\int_0^1\frac{e^{2x}+e^x}{\left(e^x+1\right)^2}dx=\ln\left(e^x+1\right)\Big|_{x=0}^{x=1}=\ln(e+1)-\ln 2=\ln\frac{e+1}{2}.$$